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Implicit Model Following and Parameter Identification of Unstable Aircraft

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A transformation in the s-plane is described which has utility in implicit model-following optimal control design application and in estimation or parameter identification problems. The objective of the transformation is, for the control problem, to achieve an unstable closed-loop system, and, for the estimation problem, to alleviate algorithm convergence problems that may arise in identifying unstable systems. For the control problem, the transformation is a shift along the real (σ) axis of the plant and model poles and zeros. This transformation is shown to be equivalent to a modified performance index but offers the advantage of compatibility with existing optimal control solution algorithms. For the estimation problem, the data are multiplied by an exponential function and the assumed measurement and process noise covariances are appropriately modified. Examples of both control and estimation applications are presented.

Nomenclature

F = $n \times n$ plant matrix
G = $n \times m$ control matrix
H = $p \times 1$ output matrix
J = performance index
L = model matrix
Q = state vector weighting matrix
q = body axis pitch rate (deg/s, rad/s)
R = control vector weighting matrix
u = $m \times 1$ control vector
u_x = velocity along body x-axis (ft/s)
v = measurement noise, white Gaussian
w = plant noise, white Gaussian
w_z = velocity along body z-axis (ft/s)
x = $n \times 1$ state vector
y = $p \times 1$ output vector
θ = pitch attitude (deg, rad)
δ_{ES} = input which produces pitching moment
δ_t = input which produces thrust
σ_T = amount of eigenvalue shift along the real axis

Introduction

THE use of variable stability aircraft for flight research experiments implies the application of control theory for configuration preparation and estimation theory for configuration validation. Unless the aircraft has independent force and moment effectors for all the degrees of freedom of interest, the control application is nontrivial because not all characteristics of the model can be perfectly matched. Similarly, unless the aircraft variable stability system is of the explicit model-following type, the estimation application is nontrivial because the achieved configuration characteristics must be determined from flight data. Accordingly, specialized control and estimation applications are frequently required during the design and conduct of an experiment.

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Examples of aircraft for which these specialized applications have been particularly important are the Calspan variable stability NT-33A and X-22A research aircraft. Both aircraft have variable stability systems of the response feedback type, but with fewer controls than degrees of freedom so that all characteristics of an arbitrary six-degree-of-freedom model cannot be matched exactly. Because an inexact match of the model is obtained, in some cases the implicit model-following variant of the linear-quadratic-optimal regulator has been found to be a useful method for computing simulation variable stability system (VSS) gains.[1] Furthermore, because the variable stability systems use response-freedback techniques, nonignorable measurement noise plus modeling errors may be present, and fairly sophisticated parameter estimation procedures have been found to be necessary for configuration validation.[2]

Recent experiments with both aircraft required simulation and parameter identification of models with unstable characteristic roots.[3,4] The previously used procedures were found to be inappropriate for this problem because:

1) The typical implicit model-following optimal regulator procedure, which is based on using the positive definite solution to a Riccati equation, will always lead to a stable closed-loop system (for state-weighting matrix non-negative definite, control weighting matrix positive definite). In particular, the closed-loop system will be stable even if the model is unstable. Hence, the desired unstable eigenvalues of the model will not be in the resulting simulation if the gains are computed in this fashion.

2) The estimation of the achieved simulation parameters from flight data is difficult because of algorithm convergence problems occasioned by the unstable dynamics.

To circumvent these difficulties, a modification to the implicit model-following optimal regulator problem, which consists of a transformation along the real axis, was made. Although developed for this application by considering the locations of the plant and model poles in the optimal solution root-square locus, the transformation is equivalent to the problem of an optimal regulator with a prescribed degree of stability as formulated by Anderson and Moore[5] and discussed by Kreindler.[6] This relationship is reviewed in this paper following the development of the transformation from the root-square locus viewpoint. The straightforward conceptual extension of the procedure to the estimation problem is also reviewed. Examples of both the control and estimation applications are given to verify the usefulness of the procedure.

σ-Shifted Transformation

Control

The implicit model-following optimal regulator problem may be formulated for the general case as follows[7]:

$$\min_u J=\int_0^\infty [(\dot{y}-Ly)^T Q(\dot{y}-Ly)+u^T Ru]\mathrm{d}t \tag{1}$$

subject to

$$\dot{x}=Fx+Gu \qquad \dot{y}=Ly \qquad y=Hx \tag{2}$$

Picking $H=I$ for simplicity (e.g., F and L same dimension), the control law that results is

$$u=-[G^TQG+R]^{-1}G^T[Q(F-L)+P]X \tag{3}$$

$$\dot{P}+P\hat{F}+\hat{F}^TP-PG\hat{R}^{-1}G^TP=-\hat{H}^T\hat{Q}\hat{H} \tag{4}$$

where

$$\hat{F}=F-G[G^TQG+R]^{-1}G^TQ(F-L) \tag{5a}$$

$$\hat{Q}=Q-QG[G^TQG+R]^{-1}G^TQ \tag{5b}$$

$$\hat{R}=G^TQG+R \tag{5c}$$

$$\hat{H}=F-L \tag{5d}$$

The closed-loop eigenvalues of $F-G[G^TQG+R]^{-1}G^T[Q(F-L)+P]$ are guaranteed to be stable for Q non-negative definite and R positive definite, which can be seen by noting that $\hat{Q}$ [Eq. (5b)] will be non-negative definite under these conditions, and $\hat{Q}\geq 0$ corresponds to the constraint required for regulators with cross terms in the performance index.[5]

A convenient technique for examining the effects of Q and R on the closed-loop eigenvalues is the root-square locus procedure.[8] For the problem discussed here Eqs. (1-5), it may be shown that the closed-loop roots are solutions to the following root-square locus expression[9]:

$$\det\{I+R^{-1}G^T[-Is-F^T]^{-1}[-Is-L^T] \times Q[Is-L][Is-F]^{-1}G\}=0 \tag{6}$$

Note that the "poles" of the locus are given by the open-loop plant poles ($\det\{Is-F\}$) and their adjoints ($\det\{-Is-F^T\}$), while the "zeros" are determined by the plant zeros *and* the model poles plus adjoints. The locus of the solutions to Eq. (6) as a function of Q and/or R is symmetric about both the σ and jw axes of the s-plane for Q non-negative definite and R positive definite. The realizable closed-loop solution for the control law given by Eq. (3) is *always* in the $\sigma\leq 0$ (left) half-plane, regardless of whether the open-loop plant and/or model roots are in the right half-plane; the $\sigma\geq 0$ half-plane is the adjoint solution. In particular, if the model pole is in the right half-plane, the realizable closed-loop locus is migrating to the *adjoint* of this pole in the left half-plane.

If, for some reason, it is desired to have the closed-loop poles approach the roots of an unstable model, it can therefore be seen that some modification to the usual procedure is required. A transformation to achieve this result is suggested by the root-square loci. If the computations could be performed for a transformed system in which the model is stable, then it may be possible to have all the adjoint zeros of the locus in the $\sigma>0$ half-plane, so that the plant roots would migrate toward the actual model roots instead of their adjoints. Transforming back to the original system then would give matching of unstable characteristics.

The requisite transformation is a shift along the σ-axis. The eigenvalues of the plant and model are given by

$$B^{-1}FB=\Lambda_p \qquad C^{-1}LC=\Lambda_m \tag{7}$$

where B, C are the eigenvector matrices for the plant and model, respectively. Choose a positive value of $\sigma(=\sigma_T)$ such that, when it is subtracted from the most unstable model eigenvalue, real or complex, the new eigenvalue would be stable. Define

$$\bar{\Lambda}_p=\Lambda_p-\sigma_T I \qquad \bar{\Lambda}_m=\Lambda_m-\sigma_T I \tag{8}$$

Then

$$B^{-1}\bar{F}B=\bar{\Lambda}_p \quad \text{if} \quad \bar{F}=F-\sigma_T I \tag{9a}$$

$$C^{-1}\bar{L}C=\bar{\Lambda}_m \quad \text{if} \quad \bar{L}=L-\sigma_T I \tag{9b}$$

Consider, therefore, a transformed optimal regulator problem:

$$\min_{\bar{u}} J=\int_0^\infty \{(\dot{\bar{y}}-\bar{L}\bar{y})^T Q(\dot{\bar{y}}-\bar{L}\bar{y})+\bar{u}^T R\bar{u}\}\mathrm{d}t \tag{10}$$

subject to

$$\dot{\bar{x}}=\bar{F}\bar{x}+G\bar{u} \qquad \dot{\bar{y}}=\bar{L}\bar{y} \qquad \bar{y}=H\bar{x} \tag{11}$$

For this problem, the actual model roots are in the left half-plane, and hence the root-square loci can approach them to the extent desired via appropriate selection of the weighting matrices.

Calling the resulting control $\bar{u}=-\bar{K}\bar{x}$, and the augmented closed-loop eigenvalues $\bar{\Lambda}_A$, we have

$$A^{-1}[\bar{F}-G\bar{K}]A=\bar{\Lambda}_A \tag{12}$$

With respect to the original system, therefore,

$$A^{-1}[F-G\bar{K}]A=\bar{\Lambda}_A+\sigma_T I=\Lambda_A \tag{13}$$

Hence, if the control design procedure made $\bar{\Lambda}_A\cong\bar{\Lambda}_m$ in the transformed system, then $\Lambda_A\simeq\Lambda_m$ as desired in the original system because $\Lambda_m=\bar{\Lambda}_m+\sigma_T I$. Figure 1 sketches the results of this procedure for an unstable model.

To show the relationship of this procedure to the previously proposed performance index,[6] we note that the variables in the original and σ-shifted systems are related via the shift theorem of Laplace transformation as

$$\bar{x}=xe^{-\sigma_T t} \quad \text{therefore} \quad \dot{\bar{x}}=\dot{x}e^{-\sigma_T t}-\sigma_T I\bar{x} \tag{14a}$$

$$\bar{y}=ye^{-\sigma_T t} \quad \text{therefore} \quad \dot{\bar{y}}=\dot{y}e^{-\sigma_T t}-\sigma_T I\bar{y} \tag{14b}$$

$$u=ue^{-\sigma_T t} \tag{14c}$$

Then

$$\dot{x}-Fx-Gu=0=e^{-\sigma_T t}(\dot{x}-Fx-Gu)$$
$$=\dot{\bar{x}}-(F-\sigma_T I)\bar{x}-G\bar{u}=\dot{\bar{x}}-\bar{F}\bar{x}-G\bar{u} \tag{15a}$$

$$\dot{\bar{y}}-\bar{L}\bar{y}=e^{-\sigma_T t}(\dot{y}-Ly) \tag{15b}$$

Therefore, the problem as posed in Eqs. (10) and (11) is, in the untransformed system,

$$\min_u J=\int_0^\infty e^{-2\sigma_T t}\{(y-Ly)^T Q(y-Ly)+u^T Ru\}\mathrm{d}t \tag{16}$$

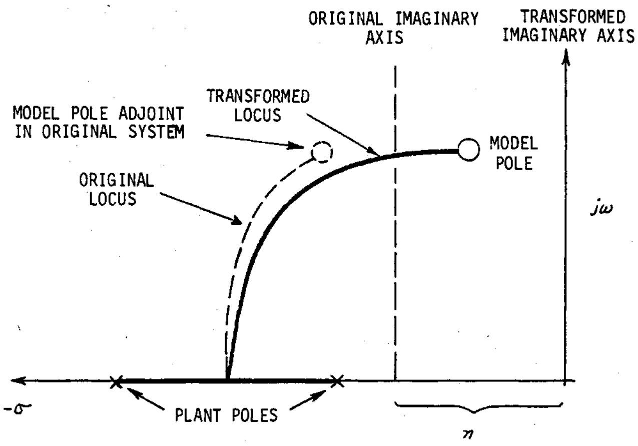

Fig. 1 Example of transformed optimal control solution.

subject to Eqs. (2). The expression in Eq. (16) is one of the specialized performance indices suggested by Kreindler and Jameson.[6] Further, it is similar in concept to the notion of prescribed stability, with only the sign of the exponent reversed.[5] The advantage of considering the transformed system [Eqs. (10) and (11)] is that no modification to existing algorithms (e.g., Riccati equation solution) is required, and that the root-square locus provides a rational means of specifying σ_T by considering the location of the most unstable model root.

Estimation/Filtering

The extension of the above concept to optimal filtering and/or estimation is straightforward. The estimation application appears to be particularly useful in parameter identification problems when the plant is unstable; in many instances, convergence of digital identification algorithms is difficult to attain for such cases because of, for example, sensitivity to initial conditions (reference trajectory).

The standard filtering problem for our application is given by

$$\dot{x} = Fx + Gu + w \quad \text{(plant)} \tag{17a}$$

$$y = Hx + v \quad \text{(measurements)} \tag{17b}$$

where

$$E\{w\} = 0 \qquad E\{v\} = 0 \tag{17c}$$

$$E\{ww^T\} = Q \tag{17d}$$

$$E\{vv^T\} = R \tag{17e}$$

$$E\{wv^T\} = 0 \tag{17f}$$

The optimal estimate of x is given by the Kalman-Bucy filter.[10] By analogy with the control problem, assume F unstable and select σ_T on the basis of the most unstable eigenvalue. Again let

$$\bar{y} = ye^{-\sigma_T t} \qquad \bar{x} = xe^{-\sigma_T t} \qquad \bar{u} = ue^{-\sigma_T t}$$

Also,

$$\bar{w} = we^{-\sigma_T t} \; \bar{v} = ve^{-\sigma_T t} \tag{18}$$

Then the σ-shifted problem becomes

$$\dot{\bar{x}} = (F - \sigma_T I)\bar{x} + G\bar{u} + \bar{w} \tag{19a}$$

$$\bar{y} = H\bar{x} + \bar{v} \tag{19b}$$

$$E\{\bar{w}\} = 0 \qquad E\{\bar{v}\} = 0 \tag{19c}$$

$$E\{\bar{w}\bar{w}^T\} = e^{-2\sigma_T t}Q \tag{19d}$$

$$E\{\bar{v}\bar{v}^T\} = e^{-2\sigma_T t}R \tag{19e}$$

$$E\{\bar{w}\bar{v}^T\} = 0 \tag{19f}$$

The solution to this problem is again the Kalman-Bucy filter, only now the process and measurement noise covariance matrices are time-varying. In terms of the initial given data, the measurements of the states (y) and inputs (u) must be multiplied by $e^{-\sigma_T t}$. Hence, the extent of the shift dictates to some extent the length of useful data record, although in theory, given sufficient resolution in the computational processes, quantities such as signal-to-noise ratio or "ratio" of covariance matrices remain constant and so no limit should exist.

For parameter identification problems in which the state model may be assumed linear and the process noise negligible, a useful parameterization of Eqs. (17) is the phase variable canonical transformation.[11]

Let

$$x = T_i z \tag{20a}$$

where T_i is a matrix whose rows are the the coefficients of the numerator polynomials of the states x to the ith controller u_i. For example, the numerators of transfer functions x_1/u_i and x_2/u_i are

$$N(x_1/u_i) = t_{1n}s^{n-1} + t_{1n-1}s^{n-2} + \ldots t_{12}s + t_{11}$$

$$N(x_2/u_i) = t_{2n}s^{n-1} + t_{2n-1}s^{n-2} + \ldots t_{22}s + t_{21} \tag{20b}$$

Then Eqs. (17a) and (17b) become

$$\dot{z} = F_0 z + G_0 u_i \tag{21a}$$

$$y = H_0 z + v \tag{21b}$$

where

$$F_0 = T_i^{-1}FT_i \qquad G_0 = T_i^{-1}G \qquad H_0 = HT_i$$

The matrices F_0 and G_0 are given by

$$F_0 = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix} \qquad G_0 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

The elements of the last row of F_0 matrix are obtained from the coefficients of the characteristic polynomial of F

$$|sI - F| = s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots a_1 s + a_0$$

With this form of the equations, the σ-shifted equation set is

$$\dot{\bar{z}} = [F_0 - \sigma_T I]\bar{z} + G_0\bar{u}_i = \bar{F}_0\bar{z} + G_0\bar{u}_i \tag{22a}$$

$$\bar{y} = H_0\bar{z} + \bar{v} \tag{22b}$$

Note that the H_0 is unaffected; the transformation moves both poles and zeros in the s-plane but does not alter their positions relative to each other. The transformed $\bar{F}_0$ results in known but nonzero diagonal terms ($\bar{f}_{0_{ii}} = -\sigma_T$, $i \neq n$) and a modification to the term representing the highest order coefficient of the characteristic equation ($\bar{f}_{0_{nn}} = -\bar{a}_{n-1}$

$=f_{0_{nn}}-\sigma_T=-a_{n-1}-\sigma_T$). By virtue of the fact that $\bar{F}_0$ provides a stable system in $\bar{z}$, however, the identification of $a_0, a_1, \ldots a_{n-1}$ appears to be facilitated.

Applications

Control

It is admitted at the outset that the control applications of the σ-shifted transformation appear currently to be limited to the problem discussed earlier: the simulation of the unstable system by another system which does not have independent control over all degrees of freedom. To illustrate how the concept works, the example concerns the simulation of the AV-8B aircraft using the X-22A aircraft.[5]

For this problem, a linearized longitudinal dynamic model of the AV-8B at several flight conditions was developed from estimated aerodynamic data. Consider the longitudinal dynamics, for which the appropriate equation of motion is

$$\begin{bmatrix} \dot{u}_x \\ \dot{w}_x \\ \dot{\theta} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} x_u & x_w & -g\cos\theta_0 & -w_0 \\ z_u & z_w & -g\sin\theta_0 & u_0 \\ 0 & 0 & 0 & 1 \\ m_u & m_w & 0 & m_q \end{bmatrix} \begin{bmatrix} u_x \\ w_x \\ \theta \\ q \end{bmatrix} + \begin{bmatrix} x_{\delta_{ES}} & x_{\delta_T} \\ z_{\delta_{ES}} & z_{\delta_T} \\ 0 & 0 \\ m_{\delta_{ES}} & m_{\delta_T} \end{bmatrix} \begin{bmatrix} \delta_{ES} \\ \delta_T \end{bmatrix} \begin{bmatrix} \alpha_0 \\ z_0 \\ 0 \\ m_0 \end{bmatrix} \tag{23}$$

For the developed AV-8B model, an example characteristic model is, at 65 knots:

$$L=\begin{bmatrix} -.044 & .01 & -32.16 & -14.15 \\ -.092 & -.24 & -1.69 & 100.53 \\ 0 & 0 & 0 & 1 \\ -.0026 & .0021 & 0 & -.191 \end{bmatrix} \tag{24}$$

The eigenvalues of L are

$$s_1=.496 \quad s_2=-.627 \quad s_{3,4}=.172\pm j\ .234 \tag{25}$$

The plant (X-22A) equations at the 65-knot flight condition are

$$F=\begin{bmatrix} -.183 & -.02 & -32.16 & -14.15 \\ -.218 & -.574 & -1.69 & 100.5 \\ 0 & 0 & 0 & 1 \\ -.0098 & -.0171 & 0 & -.145 \end{bmatrix} \tag{26a}$$

$$G=\begin{bmatrix} -.353 & .69 \\ .065 & -.89 \\ 0 & 0 \\ .319 & .024 \end{bmatrix} \tag{26b}$$

Using a shift of $\sigma_T=1.0$ and, to emphasize pitch following, weighting matrices of

$$Q=\text{diag}[1,1,1,10^5]$$

$$R=\text{diag}[.001,\quad .001]$$

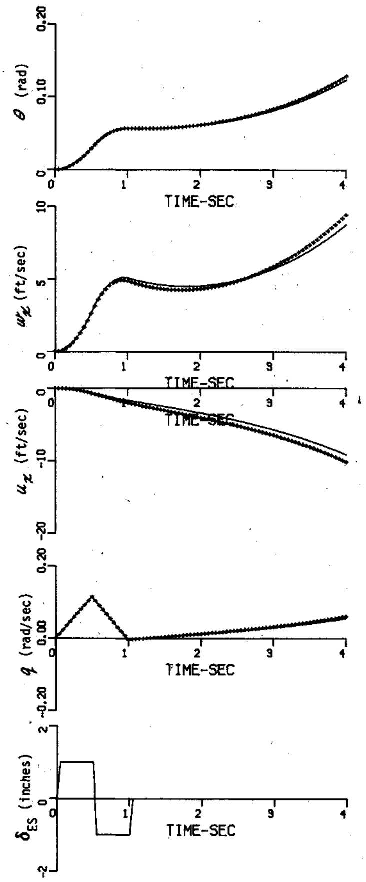

Fig. 2 Longitudinal time histories of simulation: (V=65 knots, θ_j=81 deg, δ_{ES} doublet.

the closed-loop eigenvalues of F-GK are

$$s_1=.499 \quad s_2=-.674 \quad s_{3,4}=.298\pm\ .216 \tag{27}$$

Figure 2 shows time history responses to a doublet input of the simulation. The effect of the σ-shift may be seen by letting $\sigma_T=0$ and using the same Q, R; the eigenvalues of the closed-loop system in this case are

$$s_1=-.478 \quad s_2=-.695 \quad s_{3,4}=-.314\pm j\ .242 \tag{28}$$

As was discussed previously, without the σ-shift it is not possible to match the unstable eigenvalue with conventional implicit model-following optimal control procedures.

Parameter Identification

As an example of parameter identification applications, a case from the other variable stability airplane will be used. The USAF NT-33A has a three-axis variable stability system of the same type as the X-22A. In a recent program, variations in longitudinal dynamics were examined during a landing approach task; among the configurations investigated were a few with statically unstable dynamics.[4] To identify the achieved characteristics for these cases, the σ-shifted estimation procedure was used.[11]

The equations for this identification problem are the state equation given by Eq. (23) plus the measurement equation:

$$\begin{bmatrix} u_{xm} \\ w_{xm} \\ \theta_m \\ q_m \end{bmatrix} = \begin{bmatrix} u_x \\ w_x \\ \theta \\ q \end{bmatrix} + \begin{bmatrix} u_{xb} \\ w_{xb} \\ \theta_b \\ q_b \end{bmatrix} + \begin{bmatrix} v_u \\ v_w \\ v_\theta \\ v_q \end{bmatrix} \tag{29}$$

where the subscript m means measurement, the subscript b means constant bias parameter, and the errors are assumed white noise with uncorrelated known variance. These equations were transformed to the phase variable form [Eqs.

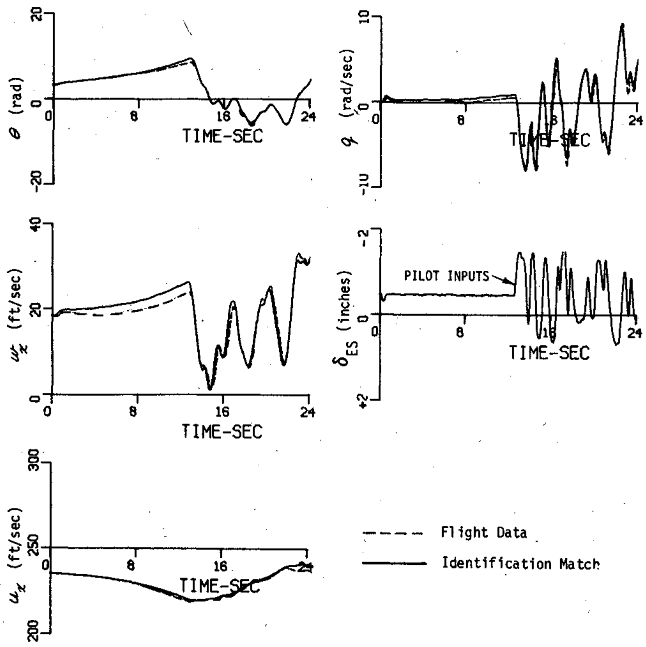

Fig. 3 Time history comparisons.

(21)]. With the transformation

$$\begin{bmatrix} u \\ w \\ \theta \\ q \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} & t_{13} & t_{14} \\ t_{21} & t_{22} & t_{23} & t_{24} \\ t_{31} & t_{32} & t_{33} & t_{34} \\ t_{41} & t_{42} & t_{43} & t_{44} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} \tag{30}$$

the transformed equations are given by

$$\begin{bmatrix} \dot z_1 \\ \dot z_2 \\ \dot z_3 \\ \dot z_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & -a_3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \delta_{ES} \tag{31a}$$

$$\begin{bmatrix} u_{xm} \\ w_{xm} \\ \theta_m \\ q_m \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} & t_{13} & t_{14} \\ t_{21} & t_{22} & t_{23} & t_{24} \\ t_{31} & t_{32} & t_{33} & t_{34} \\ t_{41} & t_{42} & t_{43} & t_{44} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} \tag{31b}$$

Since $q = \dot\theta$ and also since the numerator polynomial of θ is a second degree polynomial, it follows that $t_{31} = t_{42}$, $t_{32} = t_{43}$, $t_{33} = t_{44}$, $t_{34} = 0$, $t_{41} = 0$. F value of $\sigma_T = 0.3$ was selected [Eq. (22)] and the denominator polynomial coefficients and the elements of the transformation matrix were identified.[11] Figure 3 shows the time history comparisons of computed responses using the identified parameters with the actual flight data. The phase variable form was used for identification since the parameters of the phase variable form are transfer function coefficients and hence more closely related to the flying qualities than the parameters of Eq. (23). This configuration had one unstable root. The identified roots of the characteristic polynomial are

$$s_1 = .1194 \quad s_2 = -2.0198 \quad s_{3,4} = -.235 \pm j\,.1898$$

It is emphasized that without the shift it was not possible to achieve algorithm convergence for even this fairly simple example.

Conclusions

By considering the root-square locus representation of the linear quadratic optimal regulator problem, a transformation was used to solve the particular problem of implicit model following when the model has one or more unstable eigenvalues. It has been shown that this interpretation is equivalent to a modified performance index. The advantage of the transformation is that it permits use of existing algorithms and does not require unusual values of the weighting matrices in the performance index. At the present time, the control applications appear to be limited in scope; the author's particular problem of intentionally destabilizing an aircraft was a specialized one. The extension of the concept to parameter estimation problems should have more general applicability, however, as it has been shown to be very useful in this context; a particular application of promise is parameter estimation of unaugmented helicopters. An interesting avenue for further research would be the investigation of the effects of such a shift in explicit or combined implicit-explicit model following in terms of the magnitudes of the resulting feedback and feedforward gains.

References

[1] Lebacqz, J.V. and Aiken, E.W., "Optimal Control Techniques for V/STOL Experiment Design," *Proceedings of IEEE 1976 Conference on Decision and Control,* Clearwater Beach, Fla., Dec. 1976, pp. 1017-1022.

[2] Lebacqz, J.V., "The Efficient Application of Digital Identification Techniques to Flight Data from a Variable Stability VSTOL Aircraft," AGARD CP-172, May 1975.

[3] Lebacqz, J.V. and Radford, R.C., "An Experimental Investigation of Control-Display Requirements for a Jet-Lift VTOL Aircraft in the Terminal Area," *Proceedings of AIAA 1978 Atmospheric Flight Mechanics Conference,* Palo Alto, Calif., Aug. 1978, pp. 243-252.

[4] Smith, R.E., "Effects of Control System Dynamics on Fighter Approach and Landing Longitudinal Flying Qualities," Calspan Report No. AK-5280-F-12, Vol. 1, Calspan Corporation, Buffalo, N.Y., March 1978.

[5] Anderson, B.D.O. and Moore, J.B., *Linear Optimal Control,* Prentice-Hall, Englewood Cliffs, N.J., 1971.

[6] Kreindler, E. and Jameson, A., "Optimality of Linear Control Systems," *IEEE Transactions on Automatic Control,* Vol. Ac-17, 1972, pp. 329-351.

[7] Kreindler, E. and Rothschild, D., "Model-Following in Linear-Quadratic Optimization," *AIAA Journal,* Vol. 14, July 1976, pp. 835-842.

[8] Rynaski, E.G. and Whitbeck, R.F., "The Theory and Application of Linear Optimal Control," Calspan Report No. IH-1943-F-1, Calspan Corporation, Buffalo, N.Y., Oct. 1965.

[9] Lebacqz, J.V., "Effects of Control and Display Parameters on Piloted VTOL Decelerating Instrument Approach," Ph.D. Dissertation No. 1326-T, Princeton University, Princeton, N.J., June 1977.

[10] Kalman, R.E. and Bucy, R.S., "New Results in Linear Filtering and Prediction Theory," *Transactions of the ASME, Series D: Journal of Basic Engineering,* Vol. 83, March 1961, pp. 95-108.

[11] Govindaraj, K.S., Lebacqz, J.V., and Eulrich, B.J., "Application of Identification Algorithms to Aircraft Flight Data," *Proceedings of the Joint Automatic Control Conference,* Philadelphia, Pa., Oct. 18-20, 1978, pp. 247-258.